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LETTER TO THE EDITOR

Correlation functions of disorder operators in massive ghost theories

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Abstract
The two-dimensional ghost systems with negative integral central charge has received much attention in the last few years for their role in a number of applications and in connection with logarithmic conformal field theory. We consider the free massive bosonic and fermionic ghost systems and concentrate on the non-trivial sectors containing the disorder operators. A unified analysis of the correlation functions of such operators can be performed for ghosts and ordinary complex bosons and fermions. It turns out that these correlators depend only on the statistics although the scaling dimensions of the disorder operators change when going from the ordinary to the ghost case. As known from the study of the ordinary case, the bosonic and fermionic correlation functions are the inverse of each other and are exactly expressible through the solution of a nonlinear differential equation.

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1. Introduction

Ghost fields, namely the quantum fields violating the usual relation between spin and statistics, have been very popular in physics since when Faddeev and Popov showed their role in the quantization of non-Abelian gauge theories. In two dimensions, they have been the object of increasing interest over the last decade because of their applications in the study of disordered systems, quantum Hall states, polymer physics and dynamical models (see, e.g., [1–4]).

In the massless limit, the fermionic (anticommuting scalars) and bosonic (commuting spinors) ghost systems entering the study of these two-dimensional problems are particularly simple (free) examples of the vast class of 'non-unitary' conformal field theories which includes in particular all the conformal theories with negative central charge. The central charges of the fermionic and bosonic ghosts are $c = -2$ and $c = -1$, respectively [5], and differ only for the sign from the central charges of their counterparts with the 'right' statistics, respectively the commuting complex scalar field and the anticommuting complex spinor field. Detailed studies of the $c = -2$ and $c = -1$ ghost conformal field theories can be found in [6] and [7, 8], respectively. A comparison between the ghost systems and their counterparts with positive central charge has been performed in [9]. These models have also provided a privileged playground for logarithmic conformal field theory [10, 11].

In this letter, we consider the free *massive* bosonic and fermionic ghost systems, our interest focusing on the non-trivial sectors of these models containing the 'disorder' operators which are non-local with respect to the ghost fields. We recall that two operators $A(x)$ and $B(y)$ are said to be mutually non-local with non-locality phase $e^{2i\pi\alpha}$ if the correlation functions containing these operators pick up such a phase when $A(x)$ is taken once around $B(x)$ on the Euclidean plane. The presence of a continuous spectrum of such disorder operators in the ghost systems is expected on the same physical grounds discussed in [12] for the ordinary complex bosons and fermions. As a matter of fact, it turns out that, similarly to what is observed at the conformal level in [9], the ghost systems and the ordinary bosons and fermions are intimately related in the free massive case also. Actually, it is possible to deal in a compact form with the four cases in terms of the two parameters

$$S = \begin{cases} 1 & \text{for bosons} \\ -1 & \text{for fermions} \end{cases} \tag{1}$$

$$\varepsilon = \begin{cases} 1 & \text{for ordinary fields} \\ -1 & \text{for ghosts.} \end{cases} \tag{2}$$

In all cases the mass spectrum consists of a doublet of free particles A and $\bar{A}$ with mass m. Then, denoting by $\Phi_\alpha(x)$ the disorder operator exhibiting a non-locality phase $e^{2i\pi\alpha}$ ($e^{-2i\pi\alpha}$) with respect to (the field which interpolates) the particle A ($\bar{A}$), we will show that[4]

$$\langle \tilde{\Phi}_\alpha(x)\tilde{\Phi}_{\alpha'}(0)\rangle = e^{S\Upsilon_{\alpha,\alpha'}(m|x|)} \tag{3}$$

where $\Upsilon_{\alpha,\alpha'}(t)$ is a function expressed in terms of the solution of a nonlinear differential equation of Painlevé type. The main point to be remarked in (3) is that the rhs depends on S but not on ε, which implies that the correlation functions of the disorder operators in the bosonic and fermionic ghost systems coincide with those for the ordinary complex bosons and fermions, respectively. The latter correlators and their inversion property according to the statistics were discussed in [12–14].

The ε-independence of the rhs of (3) has to be contrasted with the fact that the nature of the operators on the lhs does depend on ε. Indeed, the values of the scaling dimensions X_α of the operators Φ_α and of the central charge in the ultraviolet limit can be written as

$$c = 2^{\delta_{S,\varepsilon}}\varepsilon \qquad X_\alpha = S\alpha(\delta_{S,\varepsilon} - \alpha). \tag{4}$$

We now turn to explaining the origin of these results.

2. Results

We work within the form factor approach in which the correlation functions are expressed as spectral series over intermediate multiparticle states after the computation of the form factors

$$f_n^\alpha(\theta_1, \ldots, \theta_n, \beta_1, \ldots, \beta_n) = \langle 0|\tilde{\Phi}_\alpha(0)|A(\theta_1)\cdots A(\theta_n)\bar{A}(\beta_1)\cdots\bar{A}(\beta_n)\rangle. \tag{5}$$

[4] We will use the notation $\tilde{\Phi}(x) \equiv \Phi(x)/\langle\Phi\rangle$ throughout this letter.

Here rapidity variables are used to parametrize the energy–momentum of a particle as $(e, p) = (m\cosh\theta, m\sinh\theta)$. The form factors can be determined in integrable quantum field theories solving a set of functional equations which in the standard cases (see, e.g., [15]) require as input the exact S-matrix (quite trivial in the free case we are dealing with) and the non-locality phases between the operators and the particles. Clearly, what we need for our present purposes is to understand how to modify these equations in order to distinguish the ghost case from that of ordinary particles discussed in [12].

The Lagrangians of the free theories we are considering contain a kinetic and a mass term, each of them linear in the fields which interpolate the particles A and $\bar{A}$. In the case of ordinary spin-statistics, Hermitian conjugation interchanges these two fields leaving the Lagrangian invariant. In the ghost case the operation made in the same way would change the sign of the Lagrangian because the terms are reordered with the 'wrong' statistics. Hence, a real Lagrangian requires that the two ghost fields are not exactly the Hermitian conjugate of each other. A suitable choice of the conjugation matrix for all cases is

$$C = \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix}. \tag{6}$$

We are now in a position to write the form factor equations which read

$$f_n^\alpha(\theta_1, \ldots, \theta_i, \theta_{i+1}, \ldots, \theta_n, \beta_1, \ldots, \beta_n) = S f_n^\alpha(\theta_1, \ldots, \theta_{i+1}, \theta_i, \ldots, \theta_n, \beta_1, \ldots, \beta_n) \tag{7}$$

$$f_n^\alpha(\theta_1 + 2\mathrm{i}\pi, \theta_2, \ldots, \theta_n, \beta_1, \ldots, \beta_n) = \varepsilon S\, \mathrm{e}^{2\mathrm{i}\pi\alpha} f_n^\alpha(\theta_1, \ldots, \theta_n, \beta_1, \ldots, \beta_n) \tag{8}$$

$$\mathrm{Res}_{\theta_1-\beta_1=\mathrm{i}\pi} f_n^\alpha(\theta_1, \ldots, \theta_n, \beta_1, \ldots, \beta_n) = \mathrm{i}S^{n-1}(1 - \mathrm{e}^{2\mathrm{i}\pi\alpha}) f_{n-1}^\alpha(\theta_2, \ldots, \theta_n, \beta_2, \ldots, \beta_n). \tag{9}$$

We work with $0 < \alpha < 1$. We see that only the presence of the factor ε in the second equation distinguishes between ghosts and ordinary particles. The origin of this factor is explained below. Shifting the rapidity of a particle by $\mathrm{i}\pi$ means inverting the sign of its energy and momentum. This inversion, together with charge conjugation, amounts to crossing the particle from the initial state to the final state. Hence, the $2\mathrm{i}\pi$ analytic continuation in equation (8) corresponds to a double crossing from the initial state to the final state and then again to the initial state, a process which produces the factor $C^2 = \varepsilon$.

The solution to the above equations can be written as

$$f_n^\alpha(\theta_1, \ldots, \theta_n, \beta_1, \ldots, \beta_n) = (-\mathrm{i})^{n\delta_{S,-\varepsilon}} S^{n(n-1)/2} (-\sin\pi\alpha)^n\, \mathrm{e}^{(\alpha - \frac{1}{2}\delta_{S,\varepsilon})\sum_{i=1}^n(\theta_i - \beta_i)}\, |A_n|_{(S)} \tag{10}$$

where A_n is an $n \times n$ matrix ($A_0 \equiv 1$) with entries

$$A_{ij} = \frac{1}{\cosh\frac{\theta_i - \beta_j}{2}} \tag{11}$$

and $|A_n|_{(S)}$ denotes the permanent[5] of A_n for $S = 1$ and the determinant of A_n for $S = -1$.

Correlation functions are obtained by inserting in between the operators a resolution of the identity in the form

$$1 = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\theta_1 \cdots \mathrm{d}\beta_n}{(n!)^2 (2\pi)^{2n}} |A(\theta_1) \cdots A(\theta_n) \bar{A}(\beta_1) \cdots \bar{A}(\beta_n)\rangle \langle \bar{A}(\beta_n) \cdots \bar{A}(\beta_1) A(\theta_n) \cdots A(\theta_1)|. \tag{12}$$

Since by crossing and Lorentz invariance we have

$$\langle \bar{A}(\beta_n) \cdots \bar{A}(\beta_1) A(\theta_n) \cdots A(\theta_1) | \tilde{\Phi}_\alpha(0) | 0 \rangle = \varepsilon^n f_n^\alpha(\beta_n + \mathrm{i}\pi, \ldots, \beta_1 + \mathrm{i}\pi, \theta_n + \mathrm{i}\pi, \ldots, \theta_1 + \mathrm{i}\pi) = \varepsilon^n f_n^\alpha(\beta_n, \ldots, \beta_1, \theta_n, \ldots, \theta_1) \tag{13}$$

[5] The permanent of a matrix differs from the determinant by the omission of the alternating sign factors $(-1)^{i+j}$.

the two-point functions take the form

$$G^{(S,\varepsilon)}_{\alpha,\alpha'}(t) = \langle \tilde{\Phi}_\alpha(x)\tilde{\Phi}_{\alpha'}(0)\rangle = \sum_{n=0}^{\infty} \frac{\varepsilon^n}{(n!)^2(2\pi)^{2n}} \int \mathrm{d}\theta_1 \cdots \mathrm{d}\theta_n \,\mathrm{d}\beta_1 \cdots \mathrm{d}\beta_n g_n^{(\alpha,\alpha')}(t|\theta_1, \ldots, \beta_n) \tag{14}$$

where

$$\begin{aligned} g_n^{(\alpha,\alpha')}(t|\theta_1, \ldots, \beta_n) &= f_n^{\alpha}(\theta_1, \ldots, \beta_n) f_n^{\alpha'}(\beta_n, \ldots, \theta_1)\, \mathrm{e}^{-te_n} \\ &= (\varepsilon S \sin\pi\alpha \sin\pi\alpha')^n\, \mathrm{e}^{(\alpha-\alpha')\sum_{i=1}^{n}(\theta_i-\beta_i)}\, |A_n|^2_{(S)}\, \mathrm{e}^{-te_n} \end{aligned} \tag{15}$$

$$t = m|x| \qquad e_n = \sum_{k=1}^{n}(\cosh\theta_k + \cosh\beta_k).$$

Hence the anticipated ε-independence of these correlators immediately follows from the cancellation between the factor ε^n contained in $g_n^{(\alpha,\alpha')}$ and that explicitly appearing in (14):

$$G^{(S,\varepsilon)}_{\alpha,\alpha'}(t) = G^{(S)}_{\alpha,\alpha'}(t). \tag{16}$$

Without repeating the discussion of [12], we recall that the spectral series for $G^{(S)}_{\alpha,\alpha'}(t)$ can be resummed in a Fredholm determinant form making the result transparent (3), namely that the bosonic and fermionic correlators are the inverse of each other. The function $\Upsilon_{\alpha,\alpha'}(t)$ is given by [13, 14]

$$\Upsilon_{\alpha,\alpha'}(t) = \frac{1}{2}\int_{t/2}^{\infty} \rho\,\mathrm{d}\rho \left[(\partial_\rho\chi)^2 - 4\sinh^2\chi - \frac{(\alpha-\alpha')^2}{\rho^2}\tanh\chi\right], \tag{17}$$

where $\chi(\rho)$ satisfies the differential equation

$$\partial_\rho^2\chi + \frac{1}{\rho}\partial_\rho\chi = 2\sinh 2\chi + \frac{(\alpha-\alpha')^2}{\rho^2}\tanh\chi(1-\tanh^2\chi) \tag{18}$$

subject to asymptotic conditions such that for $\alpha + \alpha' < 1$ one obtains

$$\lim_{t\to 0} G^{(S)}_{\alpha,\alpha'}(t) = (C_{\alpha,\alpha'}t^{2\alpha\alpha'})^{-S}. \tag{19}$$

The amplitude follows from the work of [16] and reads

$$C_{\alpha,\alpha'} = 2^{-2\alpha\alpha'} \exp\left\{2\int_0^\infty \frac{\mathrm{d}t}{t}\left[\frac{\sinh\alpha t\cosh(\alpha+\alpha')t\sinh\alpha' t}{\sinh^2 t} - \alpha\alpha'\,\mathrm{e}^{-2t}\right]\right\}. \tag{20}$$

The central charge of the ultraviolet limit and the scaling dimensions of the operators can be obtained in our off-critical framework through the sum rules [17, 18]

$$c = \frac{3}{4\pi}\int \mathrm{d}^2x\, |x|^2 \langle\Theta(x)\Theta(0)\rangle_{\text{connected}} \tag{21}$$

$$X_\alpha = -\frac{1}{2\pi}\int \mathrm{d}^2x\, \langle\Theta(x)\tilde{\Phi}_\alpha(0)\rangle_{\text{connected}} \tag{22}$$

where $\Theta(x)$ denotes the trace of the energy–momentum tensor. Since the only nonzero form factor of this operator in the free theories we are dealing with is

$$\langle 0|\Theta(0)|A(\theta)\bar{A}(\beta)\rangle = 2\pi m^2\left[-\mathrm{i}\sinh\frac{\theta-\beta}{2}\right]^{\delta_{S,-\varepsilon}} \tag{23}$$

it is easy to check that the sum rules yield the results (4).

For the discussion of the short distance behaviour of the correlators, define the exponent $\Gamma_{\alpha,\alpha'}$ through the relation

$$\langle\Phi_\alpha(x)\Phi_{\alpha'}(0)\rangle \sim |x|^{-\Gamma_{\alpha,\alpha'}} \qquad |x| \to 0. \tag{24}$$

The result (19) for $\alpha + \alpha' < 1$ follows from the operator product expansion

$$\langle\Phi_\alpha(x)\Phi_{\alpha'}(0)\rangle \sim |x|^{X_{\alpha+\alpha'}-X_\alpha-X_{\alpha'}}\langle\Phi_{\alpha+\alpha'}\rangle + \cdots. \tag{25}$$

The ε-dependence of the scaling dimensions in (4) affects only the term linear in α and cancels out in the above combination leaving

$$\Gamma_{\alpha,\alpha'} = 2S\alpha\alpha' \qquad 0 < \alpha + \alpha' < 1. \tag{26}$$

It seems more difficult to give a unified description for the range $1 < \alpha + \alpha' < 2$. On the basis of the discussion of [12] we expect that for $S = \varepsilon$ the short distance behaviour (25) still holds provided $\alpha + \alpha'$ is taken modulo 1. Then one finds

$$\Gamma_{\alpha,\alpha'} = 2S[\alpha\alpha' + 1 - (\alpha + \alpha')] \qquad 1 < \alpha + \alpha' < 2. \tag{27}$$

This result is recovered in the case $S = -1$, $\varepsilon = 1$ due to the fact that the first-order off-critical correction becomes leading in this range of $\alpha + \alpha'$ [12]. The mechanism that should lead to (27) in the remaining case of the bosonic ghost is not clear to us at present.

At the border value $\alpha + \alpha' = 1$ the correlators develop a logarithmic correction that is most easily evaluated for the well-studied case of ordinary complex fermions [16]. One concludes

$$\lim_{t\to 0} G^{(S)}_{\alpha,1-\alpha}(t) = [\mathcal{B}_\alpha t^{2\alpha(1-\alpha)} \ln(1/t)]^{-S} \tag{28}$$

with

$$\mathcal{B}_\alpha = 2^{1-2\alpha(1-\alpha)}\,\mathrm{e}^{-(I_\alpha + I_{1-\alpha})} \tag{29}$$

$$I_\alpha = \int_0^\infty \frac{\mathrm{d}t}{t}\left(\frac{\sinh^2 \alpha t}{\sinh^2 t} - \alpha^2\,\mathrm{e}^{-2t}\right). \tag{30}$$

An interesting check of our results for the ghost correlation functions can be performed for the operator $\Phi_{1/2}$ in the fermionic ghost theory. In fact, the free massive fermionic ghost can formally be regarded as a limit of the $\varphi_{1,3}$ perturbation of the minimal conformal models with central charge [19]

$$c = 1 - \frac{6}{p(p+1)}, \tag{31}$$

possessing the spectrum of scalar primary fields $\varphi_{l,k}$ with scaling dimensions

$$X_{l,k} = \frac{((p+1)l - pk)^2 - 1}{2p(p+1)}. \tag{32}$$

The required values $c = -2$ and $X_{1,3} = 0$ are found as[6] $p \to 1$. Our operator $\Phi_{1/2}$ with scaling dimension $-1/4$ is identified with $\varphi_{1,2}$. From the operator product expansion of the $\varphi_{l,k}$ we have for $p \to 1$

$$\begin{aligned}\langle\tilde\varphi_{1,2}(x)\tilde\varphi_{1,2}(0)\rangle &\simeq \frac{|x|^{-2X_{1,2}}}{\langle\varphi_{1,2}\rangle^2}(1 + C\langle\varphi_{1,3}\rangle|x|^{X_{1,3}})\\ &\simeq \frac{|x|^{1/2}}{\langle\varphi_{1,2}\rangle^2}\{1 + C\langle\varphi_{1,3}\rangle[1 + (p-1)\ln|x|]\}.\end{aligned} \tag{33}$$

It can be checked from the known values of the structure constant C [20] and of the vacuum expectation values in $\varphi_{1,3}$-perturbed minimal models [16] that $C\langle\varphi_{1,3}\rangle = -1$ and $(p-1)/\langle\varphi_{1,2}\rangle^2 = \mathcal{B}_{1/2} m^{1/2}$ as $p \to 1$, so that the result (28) with $S = -1$ and $\alpha = 1/2$ is indeed recovered.

[6] The genuine minimal models of the series (31) have $p = 3, 4, \ldots$. It is known, however, that many results can be extended to continuous values of p.

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